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UNITED STATES PATENT APPLICATION

OF

SILVIU BORAC

FOR

**COMPUTER GRAPHICS SYSTEM AND COMPUTER-IMPLEMENTED METHOD FOR GENERATING
SMOOTH FEATURE LINES FOR SUBDIVISION SURFACES**

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1

FIELD OF THE INVENTION

2 The invention relates generally to the field of computer graphics, and more specifically to
3 generation of smooth feature lines on subdivision surfaces representing surfaces of objects.

4

BACKGROUND OF THE INVENTION

5 Generally, in computer graphics, objects are represented as surfaces, with the surfaces being
6 represented by meshes at a particular level. A mesh at a particular consists of a set of vertices, or
7 points in three-dimensional space, which are interconnected by edges. The edges define polygonal
8 faces, which may be in the form of triangles, quadrilaterals, and so forth. In some computer graphic
9 operations, it is desirable to generate a representation of a surface at a finer resolution than a current
10 representation. There are several popular methodologies for generating a representation of a surface
11 at a finer resolution than a current representation, including a Catmull-Clark surface subdivision
12 methodology, which is used in connection with surfaces defined by a quadrilateral mesh, and a Loop
13 surface subdivision methodology, which is used in connection with surface defined by a triangular
14 mesh. Generally, both methodologies make use of respective subdivision rules at respective vertices
15 defining the surface at a particular level in the mesh to generate a mesh in a next higher subdivision
16 level. The surface of the respective object, which is referred to as the "subdivision surface" or "limit
17 surface," is taken as being defined by a mesh as the subdivision level approaches infinity.

18 A feature on a subdivision surface can be defined by a feature line in the mesh that defines
19 the respective surface. A feature line can be in the form of a sharp crease or a smooth curve. For
20 a smooth feature line, a normal vector, which is the vector that is perpendicular to a plane that is
21 tangent to the surface, will vary continuously across the smooth feature line. On the other hand, for
22 a sharp crease, the normal vector varies discontinuously across the crease, and in fact is not defined
23 at the crease. However, a definition for a smooth curve can be derived from a definition for a sharp
24 crease using one or more parameters that are used to define the sharpness of the curve across vertices
25 in the mesh at a particular subdivision level. D. Zorin, "Stationary Subdivision And Multi-
26 Resolution Surface Representation," Ph.D. Thesis, California Institute of Technology, Pasadena, CA,

1 1998, describes a methodology for generating a smooth feature line using parameters, but the
2 methodology described there results in surfaces of relatively low quality, even in surfaces that are
3 not smooth at some vertices in the surface topology. T. DeRose, et al., "Subdivision surfaces in
4 character animation," SIGGRAPH 98 Conference Proceedings, Annual Conference Series, pages
5 85-94, ACM SIGGRAPH, 1998, describes a methodology for generating a smooth feature line based
6 on applying a subdivision rule corresponding to sharp creases, as described in H. Hoppe, et al.,
7 "Piecewise Smooth Surface reconstruction," SIGGRAPH 94 Conference Proceedings, Annual
8 Conference Series, ACM SIGGRAPH, 1994, up to a selected fineness level, and a rule
9 corresponding to smooth interior points thereafter. Applying the two distinctly different types of
rules makes efficient evaluation of the resulting surface difficult.

SUMMARY OF THE INVENTION

1 The invention provides a new and improved system and method for generating a smooth
2 feature line in a subdivision surface.

3 In brief summary, the invention provides an arrangement for generating a representation of
4 a feature in a surface defined by a mesh representation, the mesh comprising at a selected level a
5 plurality of points including at least one point connected to a plurality of neighboring points by
6 respective edges, the feature being defined in connection with the vertex and at least one of the
7 neighboring points and the edge interconnecting the vertex and the at least one of the neighboring
8 points in the mesh representation. The feature generating arrangement comprises a weight vector
9 generator module and a feature representation generator module. The weight vector generator
10 module is configured to generate at least one weight vector based on a parameterized subdivision
11 rule defined at a plurality of levels, for which a value of at least one parameter differs at at least two
12 levels in the mesh. The feature representation generator module configured to use the at least one
13 weight vector and positions of the vertex and the neighboring points to generate the representation
14 of the feature.

15 One embodiment of the invention comprises an arrangement for generating a smooth feature
16 line in the subdivision surface.

1

BRIEF DESCRIPTION OF THE DRAWINGS

2 This invention is pointed out with particularity in the appended claims. The above and
3 further advantages of this invention may be better understood by referring to the following
4 description taken in conjunction with the accompanying drawings, in which:

5 FIG. 1 depicts a computer graphics system including an arrangement for generating a smooth
6 feature line in a subdivision surface, constructed in accordance with the invention;

7 FIG. 2 depicts a mesh representing a surface to which the Loop surface subdivision
8 methodology has been applied;

9 FIG. 3 depicts a stencil useful in understanding the Loop surface subdivision methodology;

10 FIGS. 4A through 4E depict a mesh representing a surface to which the Catmull-Clark
11 surface subdivision methodology has been applied;

12 FIG. 5 depicts a stencil useful in understanding the Catmull-Clark surface subdivision
13 methodology;

14 FIG. 6 depicts a stencil useful in understanding a methodology for generating a sharp crease
15 on a subdivision surface;

16 FIG. 7 depicts a stencil useful in understanding a methodology for generating a smooth
17 feature line used in connection with subdivision surfaces defined by triangular meshes;

18 FIGS. 8 and 9 depict tables of coefficients used in the methodology used in connection with
19 surfaces defined by triangular meshes described in connection with FIG. 7;

20 FIG. 10 depicts a flow chart describing operations performed by the computer graphics
21 system in connection with the methodology used in connection with surfaces defined by triangular
22 meshes described in connection with FIG. 7;

23 FIG. 11 depicts a stencil useful in understanding a methodology for generating a smooth
24 feature line used in connection with subdivision surfaces defined by quadrilateral meshes;

1 FIGS. 12 and 13 depict tables of coefficients used in the methodology used in connection
2 with surfaces defined by quadrilateral meshes described in connection with FIG. 11.

3 **DETAILED DESCRIPTION OF AN ILLUSTRATIVE EMBODIMENT**

4 FIG. 1 depicts a computer graphics system 10 including an arrangement for generating a
5 smooth feature line in connection with a subdivision surface, constructed in accordance with the
6 invention. With reference to FIG. 1, the computer graphics system includes a processor module 11,
7 one or more operator input devices 12 and one or more display devices 13. The display device(s)
8 13 will typically comprise a frame buffer, video display terminal or the like, which will display
9 information in textual and/or graphical form on a display screen to the operator. The operator input
10 devices 12 for a computer graphics system 10 will typically include a pen 14 which is typically used
11 in conjunction with a digitizing tablet 15, and a trackball or mouse device 16. Generally, the pen
12 14 and digitizing tablet will be used by the operator in several modes. In one mode, the pen 14 and
13 digitizing tablet are used to provide updated shading information to the computer graphics system.
14 In other modes, the pen and digitizing tablet are used by the operator to input conventional computer
15 graphics information, such as line drawing for, for example, surface trimming and other information,
16 to the computer graphics system 10, thereby to enable the system 10 to perform conventional
17 computer graphics operations. The trackball or mouse device 16 can be used to move a cursor or
18 pointer over the screen to particular points in the image at which the operator can provide input with
19 the pen and digitizing tablet. The computer graphics system 10 may also include a keyboard (not
20 shown) which the operator can use to provide textual input to the system 10.

21 The processor module 11 generally includes a processor, which may be in the form of one
22 or more microprocessors, a main memory, and will generally include one a mass storage subsystem
23 including one or more disk storage devices. The memory and disk storage devices will generally
24 store data and programs (collectively, "information") to be processed by the processor, and will store
25 processed data which has been generated by the processor. The processor module includes
26 connections to the operator input device(s) 12 and the display device(s) 13, and will receive
27 information input by the operator through the operator input device(s) 12, process the input
28 information, store the processed information in the memory and/or mass storage subsystem. In

1 addition, the processor module can provide video display information, which can form part of the
2 information obtained from the memory and disk storage device as well as processed data generated
3 thereby, to the display device(s) for display to the operator. The processor module 11 may also
4 include connections (not shown) to hardcopy output devices such as printers for facilitating the
5 generation of hardcopy output, modems and/or network interfaces (also not shown) for connecting
6 the system 10 to the public telephony system and/or in a computer network for facilitating the
7 transfer of information, and the like.

8 The invention provides an arrangement for generating a smooth feature line in a three-
9 dimensional subdivision surface. The subdivision surface, in turn, is defined by control points that
10 are interconnected by edges to form a three-dimensional mesh. Generally, a subdivision surface is
11 initially defined by a mesh at a particular degree of granularity or fineness. Using one of several
12 methodologies, the mesh can be refined through a series of levels of increasing subdivision levels,
13 with the subdivision surface being the limit as the subdivision level approaches infinity. In the
14 following, the each mesh level will be identified an index, with one mesh level being identified by
15 index "j" and the next higher subdivision level mesh being identified by index "j+1." The
16 subdivision surface is essentially defined by the positions of the surface's "limit points," which are
17 the essentially the control points as the subdivision level approaches infinity, and the orientations
18 of the associated normal vectors. Each normal vector defines the orientation of a tangent plane at
19 the respective point on the subdivision surface and, in turn, can be defined by the cross product of
20 vectors that are tangent to the subdivision surface at the respective point.

21 Before describing the operations performed by the smooth feature line generating
22 arrangement, it would be helpful to describe two surface subdivision methodologies, namely the
23 aforementioned Loop surface subdivision methodology and a Catmull-Clark surface subdivision
24 methodology, with which the arrangement is used..

25 Loop's surface subdivision methodology will be described in connection with FIGS. 2 and
26 3. FIG. 2 depicts a portion of an illustrative surface defined by a mesh, and a subdivision surface
27 derived therefrom, and FIG. 3 depicts a stencil useful in understanding the surface subdivision
28 methodology. Generally, in Loop's surface subdivision methodology, each triangular face in the
29 original mesh is split into a plurality of subfaces, the subfaces defining the next higher subdivision

level mesh. The vertices of the next higher subdivision level mesh are positioned using weighted averages of the positions of the vertices in the original mesh. More specifically, and with reference to FIG. 2, a mesh 20 at a "j" subdivision level is depicted that includes a vertex v_q (vertex $v_q=v_q(0)$) 21(0) located at position $c^j(q)$, and a plurality "K" of surrounding points $v_q(1)$ through $v_q(K)$ (generally, $v_q(k)$) 21(1) through 21(K) (generally identified by reference numeral 21(k)) located at positions at respective positions $c^j(1)$ through $c^j(K)$, the points $v_q(k)$ 21(k) being those points in the mesh 20 that are connected to vertex v_q by edges. ("K," the number of points connected to vertex v_q will sometimes be referred to as the vertex's "valence.") The stencil depicted in FIG. 3 is useful in understanding the ordering of the index "k" for the points surrounding the respective vertex v_q . Denoting the set of the indices of the points $v_q(k)$ in the mesh that are connected to vertex v_q by $N(q,j)$, a mesh at the next higher subdivision level "j+1" corresponds to vertex v'_q 22(0) located at position $c^{j+1}(q)$ and a set of surrounding points $v'_q(l)$ 22(l) at located at positions $c^{j+1}(l)$ and connected to the vertex v'_q 22(0). The higher subdivision level mesh is constructed by providing the vertex v'_q 22(0) at a position $c^{j+1}(q)$ that is determined by

$$c^{j+1}(q) = (1 - a(K)) c^j(q) + \frac{a(K)}{K} \sum_{k \in N(q,j)} c^j(k) \quad (1)$$

and points $v'_q(l)$ 22(l) at positions $c^{j+1}(l)$ that are determined by

$$c^{j+1}(l) = \frac{3}{8} [c^j(q) + c^j(k)] + \frac{1}{8} [c^j(k-1) + c^j(k+1)], \quad k = 1, \dots, K \quad (2),$$

where the weighting factor $a(K)$ is given by

$$a(K) = \frac{5}{8} - \left(\frac{3 + 2 \cos\left(\frac{2\pi}{K}\right)}{8} \right)^2 \quad (3).$$

1 Equations (1) and (2) can be summarized in a single equation defining a subdivision rule using a
 2 subdivision matrix $S_{T,K}$ (where index "T" refers to "Triangular") as follows

3 $c^{J+1} = S_{T,K} c^J$ (4),

4 which may be written component-wise as

5 $(c^{J+1})_l = \sum_m (S_{T,K})_{l,m} (c^J)_m$ (5),

6 where

7 $(S_{T,K})_{l,m} = \begin{cases} 1 - a(K) & \text{if } l = 0, m = 0 \\ \frac{a(K)}{2} & \text{if } l = 0, m = 1, \dots, K \\ \frac{3}{8} & \text{if } l = 1, \dots, K, m = 0 \\ & \text{or } l = m = 1, \dots, K \\ \frac{1}{8} & \text{if } l = 2, \dots, K, m = l - 1 \\ & \text{or } l = 1, m = K \\ & \text{or } l = 1, \dots, K, m = l + 1 \\ & \text{or } l = K, m = 1 \\ 0 & \text{otherwise} \end{cases}$ (6),

8 where indices "l" and "m" range from "zero" to, and including, "K."

9 Equations (1) and (2), or equivalently equations (4) through (6), are applied by selecting each
 10 point in the mesh at subdivision level "j" as a vertex, to provide points for the mesh at the
 11 subdivision level "j+1." Loop's surface subdivision methodology can be applied recursively to
 12 provide a mesh at any desired subdivision level. It will be appreciated that equation (2) will provide

1 that the point $v'_q(l)$ 22(l) that is created when the methodology is applied to point $v_q(0)$ as the
2 vertex will be in the same position when the methodology is applied to point $v_q(k)$ 21(k) as the
3 vertex.

4 It will be appreciated that, the Loop surface subdivision methodology provides

5 (i) one new point (reference points $v'_q(l)$ described above), at location $c^{j+1}(l)$ in the next higher
6 ("j+1") subdivision level mesh, for, and generally somewhere between, each pair of points in the
7 level "j" mesh, and

8 (ii) a relocated point (reference vertex v'_q described above), at location $c^{j+1}(q)$ in the "j+1"
9 level mesh for, and generally somewhat near, each point in the "j" level mesh,
10 which are interconnected to form the "j+1" level mesh.

11 The Catmull-Clark subdivision methodology produces smooth surfaces using a small number
12 of neighboring vertices. The Catmull-Clark surface subdivision methodology will be described in
13 connection with FIGS. 4A through 4E and 5. FIG. 4A depicts an illustrative surface defined by a
14 mesh, and FIGS. 4B through 4E are useful in understanding operations performed in generating a
15 subdivision surface derived from the surface depicted in FIG. 4A. FIG. 5 depicts a stencil useful in
16 understanding the surface subdivision methodology, and in particular is useful in understanding the
17 ordering of the index "k" for the points surrounding the vertex v_q (vertex $v_q=v_q(0)$) around which the
18 surface subdivision methodology is to be performed. With reference to FIG. 4A, that FIG. depicts
19 a mesh 30 consisting of four quadrilaterals 31(1) through 31(4). Each quadrilateral is referred to as
20 a face. The quadrilaterals are defined by points 32(0), which is common to all of the quadrilaterals
21 31(1) through 31(4), and other points 32(1) through 32(8). Generally, the Catmull-Clark surface
22 subdivision methodology is performed in a series of iterations, including

23 (i) a face point generation iteration, in which, for each face, a face point 33(1) through 33(4)
24 is generated as the average of the points 32(p) defining the respective face (reference FIG. 4B);

25 (ii) an edge point generation iteration in which, for each edge, a new edge point 34(1)
26 through 34(4) is generated as the average of the midpoint of the original edge with the two new face

1 points of the faces adjacent to the edge (reference FIG. 4C; the face points generated during the face
2 point generation iteration are shown as circles);

3 (iii) a vertex point iteration in which a vertex point 35 is generated in relation to the positions
4 of the new face points generated in iteration (i) for faces that are adjacent to the original vertex point,
5 the positions of the midpoints of the original edges that are incident on the original vertex point, the
6 position of the original vertex point, and the valence of the original vertex point (reference FIG. 4D;
7 the face and edge points generated during the face point and edge point generation iterations are
8 shown as circles); and

9 (iv) a mesh connection step (reference FIG. 4E) in which:

10 (a) each new face point is connected to the edge points of the edges defining the original
11 face; and

12 (b) each new vertex point is connected to the new edge points of all original edges
13 incident on the original vertex point.

14 More specifically, in the Catmull-Clark subdivision methodology, denoting, for a vertex v_q
15 at location $c^j(q)$,

16 (i) the set of indices of first-order neighboring points $c^j(l)$ that are connected thereto (for
17 example, for point 32(0), point 32(2) and 32(8)) by $N_e(q,j)$, and

18 (ii) the set of indices of second-order neighboring points that are opposite to vertex $c^j(q)$ with
19 respect to the level "j" faces (for example, for point 32(0), point 32(1)) that are incident with vertex
20 $c^j(q)$ by $N_f(q,j)$

21 a mesh at the next higher subdivision level "j+1" is constructed as follows. In the face point
22 generation iteration, the face points are generated and located at positions $c^{j+1}(m_i)$ determined as
23 follows

$$\begin{aligned}
c^{j+1}(m_i) &= \frac{1}{4} (c^j(q) + c^j(l_i) + c^j(l_{i+1}) + c^j(r_i)), \\
m_i &\in N_f(q, j+1), l_i, l_{i+1} \in N_e(q, j), r_i \in N_f(q, j)
\end{aligned} \tag{7}$$

1

2 In the edge point generation iteration, the edge points are generated and located at positions $c^{j+1}(l_i)$
 3 determined as follows:

$$\begin{aligned}
c^{j+1}(l_i) &= \frac{1}{4} (c^j(q) + c^j(k_i) + c^{j+1}(m_{i-1}) + c^{j+1}(m_i)), \\
l_i &\in N_e(q, j+1), k_i \in N_e(q, j), m_{i-1}, m_i \in N_f(q, j+1)
\end{aligned} \tag{8}$$

5 In the vertex point generation iteration, the new vertex points are generated and located at positions
 6 as follows:

$$\begin{aligned}
c^{j+1}(q) &= \frac{K-2}{K} c^j(q) + \frac{1}{K^2} \sum_{i=0}^{K-1} c^j(l_i) + \frac{1}{K^2} \sum_{i=0}^{K-1} c^{j+1}(m_i) \\
l_i &\in N_e(j+1, q), m_i \in N_f(j+1, q)
\end{aligned} \tag{9}$$

8 In terms of only points c^j , that is substituting for the last term in equation (9),

$$c^{j+1}(q) = \left(1 - \frac{7}{4K}\right) c^j(q) + \frac{3}{2} \frac{1}{K^2} \sum_{l \in N_e(q, j)} c^j(l) + \frac{1}{4K^2} \sum_{m \in N_f(q, j)} c^j(m) \tag{10}$$

9

1 Generally, for quadrilateral meshes, the arrangement makes use of the Catmull-Clark
 2 methodology, except for the case of K=3 in equation (9). In that case, the arrangement makes use
 3 of

4

$$c^{j+1}(q) = (1 - 8\gamma) c^j(q) + 4\gamma \frac{1}{K} \sum_{l \in N_e(q,j)} c^j(l) + 4\gamma \frac{1}{K} \sum_{m \in N_f(q,j+1)} c^{j+1}(m) \quad (11),$$

5 where $\gamma = \frac{3}{38}$. Equation (11) would correspond to the Catmull-Clark methodology (equation (9))

6 with $\gamma = \frac{1}{4K}$, except that, for K=3, $\gamma = \frac{3}{38}$ instead of $\frac{1}{12}$. In terms of only points c^j (compare
 7 equation (10))

8

$$c^{j+1}(q) = (1 - 7\gamma) c^j(q) + 6\gamma \frac{1}{K} \sum_{l \in N_e(q,j)} c^j(l) + \gamma \frac{1}{K} \sum_{m \in N_f(q,j)} c^j(m) \quad (12).$$

9 These equations can be summarized by a single equation defining a subdivision rule using the
 10 subdivision matrix $S_{Q,K}$ (where index "Q" refers to "Quadrilateral") as follows:

11

$$c^{j+1} = S_{Q,K} c^j \quad (13),$$

12 which may be written component-wise as

13

$$(c^{j+1})_l = \sum_m (S_{Q,K})_{l,m} (c^j)_m \quad (14),$$

14 where

$$\begin{aligned}
 (S_{Q,K})_{l,m} = & \begin{cases}
 1 - \frac{7}{4K} & \text{if } l = 0, m = 0 \\
 \frac{3}{2} \frac{1}{K^2} & \text{if } l = 0, m = 1, \dots, K \\
 \frac{1}{K^2} & \text{if } l = 0, m = K + 1, \dots, 2K \\
 \frac{3}{8} & \text{if } l = 1, \dots, K, m = 0 \text{ or } l \\
 \frac{1}{16} & \text{if } l = 1, m = K \\
 & \text{or } l = 2, \dots, K, m = l - 1 \\
 & \text{or } l = 1, \dots, K - 1, m = l + 1 \\
 & \text{or } l = K, m = 1 \\
 & \text{or } l = 1, m = 2K \\
 & \text{or } l = 2, \dots, K, m = K + l - 1 \\
 & \text{or } l = 1, \dots, K, m = K + 1 \\
 \frac{1}{4} & \text{if } l = K + 1, \dots, 2K, m = 0, l - K, l - K + 1, \text{ or } l \\
 & \text{or } l = 2K, m = 0, K, 1 \text{ or } 2K \\
 0 & \text{otherwise}
 \end{cases} \quad (15),
 \end{aligned}$$

1

2 where indices "l" and "m" range from "zero" to, and including, "2K."

3 As is apparent from the above discussion, the Catmull-Clark surface subdivision
 4 methodology applied to a quadrilateral mesh at one "j-th" subdivision level will produce a
 5 quadrilateral mesh at the next higher "j+1-st" subdivision level having the same number of points
 6 as the mesh at the "j-th" subdivision level, but the points in the higher ("j+1") subdivision level may
 7 be at different positions than in the "j-th" level. Equations (7) through (12), or equivalently
 8 equations (13) through (15), are applied by selecting each point in the mesh at subdivision level "j"
 9 as a vertex, to provide points for the mesh at subdivision level "j+1." The Catmull-Clark surface

1 subdivision methodology can be applied recursively to provide a mesh at any desired subdivision
2 level.

3 Sharp creases in a subdivision surface may be generated by local modifications of the surface
4 subdivision equations described above (equations (6) through (8) in the case of a triangular mesh,
5 equations (13) through (15) in the case of a quadrilateral mesh). This will be illustrated in
6 connection with FIG. 6, which depicts a stencil that is useful in understanding the ordering of the
7 index "k" for the points on opposing sides of a respective vertex v_q (vertex $v_q = v_q(0)$) through which
8 a crease is to occur. By marking the edges of the mesh at which a crease is to occur, chains of edges
9 can be defined that will define the position of a crease on a subdivision surface. As in the cases of
10 both triangular and quadrilateral meshes, the positions c^{j+1} of vertices and points in the higher
11 "j+1-st" subdivision level can be determined from positions of vertices and points c^j in the "j-th"
12 level using a defining a subdivision rule comprising a subdivision matrix S_C (where index "C" refers
13 to "crease") as follows:

$$c^{j+1} = S_C c^j \quad (16),$$

15 which may be written component-wise as

$$(c^{j+1})_l = \sum_m (S_C)_{l,m} (c^j)_m \quad (17),$$

17 where

$$(S_C)_{l,m} = \begin{cases} \frac{3}{4} & \text{if } l = 0, m = 0 \\ \frac{1}{8} & \text{if } l = 0, m = 1 \text{ or } 2 \\ \frac{1}{2} & \text{if } l = 1 \text{ or } 2, m = 0 \text{ or } l \\ 0 & \text{otherwise} \end{cases} \quad (18),$$

1 where indices "1" and "m" range from "zero" to, and including, "two." This subdivision rule can be
2 used for creases in the cases subdivision surfaces defined by both triangular meshes and quadrilateral
3 meshes.

4 The invention provides a system and method for generating smooth feature lines in
5 subdivision surfaces. As with a sharp crease, a smooth feature line is defined by marking a series
6 of edges at a given subdivision level, which edges are to define the feature line. The lowest
7 subdivision level at which the edges that are associated with a smooth feature line are marked will
8 be referred to as the definition level " j_D " for the smooth feature line. In addition, a sharpness
9 parameter will be associated with each marked edge in the definition level j_D for the smooth feature
10 line. In one embodiment, the value of the sharpness parameter is in the form of a real number in the
11 interval $[0,1]$, that is, in the interval between "zero" and "one," inclusive of the endpoints. If an edge
12 has a sharpness parameter with the value "zero," there is no crease along the edge. On the other
13 hand, if an edge has a sharpness parameter with the value "one," the edge forms part of a sharp
14 crease as described above.

15 Generally, a smooth feature is obtained by using a subdivision rule that generalizes the
16 subdivision rules described above (reference equations (4) through (6) in the case of a triangular
17 mesh and equations (13) through (15) in the case of a quadrilateral mesh). The subdivision rule
18 makes use of two parameters s_1 and s_2 which are the respective values of the sharpness parameter
19 on two sides of a vertex across which a smooth feature line is to be generated. A subdivision
20 surface's limit surface, that is, the surface defined by the subdivision surface's limit points, is smooth
21 in the neighborhood of a smooth feature line. That is, for each point on the limit surface that is
22 associated with a smooth feature line, that is, for each point for which the values of the sharpness
23 parameters on respective sides of the point are other than "one," the orientation of the vector that is
24 normal to the limit surface will vary continuously. However, for the limit surface at a point for
25 which the sharpness parameter have the value is "one," which will be the case at a sharp crease, the
26 normal vector will not be defined, since the orientation of the normal vector on respective sides of
27 the point will differ.

28 Operations performed by the computer graphics system 10 in connection with generating a
29 smooth feature line will be described in connection with FIGS. 7 through 10, in the case of a

1 subdivision surface defined by a triangular mesh, and in connection with FIGS. 11 through 14, in
2 the case of a subdivision surface defined by a quadrilateral mesh. In both cases, the smooth feature
3 line will be defined by the positions of limit points, as well as the normal vectors at the respective
4 limit point, in a region of a subdivision surface that is proximate the line corresponding to the edges
5 that have been marked. As noted above, the normal vectors correspond to the cross product of two
6 tangent vectors at the respective limit point, namely, a tangent vector along the smooth feature line
7 at the respective limit point and a tangent vector across the smooth feature line at the respective limit
8 point.

9 With reference to FIG. 7, that FIG. depicts a portion of a subdivision surface defined by a
10 triangular mesh with vertex v_q (vertex $v_q=v_q(0)$) and points $v_q(1)$ through $v_q(K)$ with indices
11 numbered as depicted in the FIG. The vertices are labeled such that a crease, which is to define part
12 of a smooth feature line, passes through point $v_q(1)$, vertex v_q and point $v_q(L+1)$ along edges $v_q(1), v_q$
13 (which will be referred to as edge (1,0)) and $v_q, v_q(L+1)$ (which will be referred to as edge (0,L+1)).
14 The value of the sharpness parameter associated with the edge (1,0) will be referred to as s_1 and the
15 value of the sharpness parameter associated with the edge (0,L+1) will be referred to as s_2 . Since
16 edges (1,0) and (0,L+1) have been indicated as being edges that define a crease, the value of the
17 sharpness parameter will be non-zero. The values of the sharpness parameters associated with the
18 other edges that are on the vertex v_q may be "zero," or non-zero. Generally the subdivision rule
19 described in equation (4) (or, equivalently, equation (5)) can be used to determine the positions of
20 respective vertices, using the subdivision matrix

$$\left(S_{sc,T,K,L}(s_1, s_2) \right)_{l,m} = \begin{cases}
 \left(1 - s_3\right)\left(1 - a(K)\right) + \frac{3}{4}s_3 & \text{if } l = 0, m = 0 \\
 \left(1 - s_3\right)\frac{a(K)}{K} + \frac{1}{8}s_3 & \text{if } l = 0, m = 1 \text{ or } L + 1 \\
 \left(1 - s_3\right)\frac{a(K)}{K} & \text{if } l = 0, m = 2, \dots, L \\
 & \text{or } l = 0, m = L + 2, \dots, K \\
 \frac{3}{8} + \frac{1}{8}s_2 & \text{if } l = 1, m = 0 \text{ or } 1 \\
 \frac{3}{8} + \frac{1}{8}s_1 & \text{if } l = L + 1, m = 0 \text{ or } L + 1 \\
 \frac{1}{8}\left(1 - s_2\right) & \text{if } l = 1, m = 2 \text{ or } K \\
 \frac{1}{8}\left(1 - s_1\right) & \text{if } l = L + 1, m = L \text{ or } L + 2 \\
 \frac{3}{8} & \text{if } l = 2, \dots, L, m = 0 \\
 & \text{or } l = L + 2, \dots, K, m = 0 \\
 & \text{or } l = m = 2, \dots, L \\
 & \text{or } l = m = L + 2, \dots, K \\
 \frac{1}{8} & \text{if } l = 2, \dots, L, m = l - 1 \\
 & \text{or } l = L + 2, \dots, K, m = l - 1 \\
 & \text{or } l = 2, \dots, L, m = l + 1 \\
 & \text{or } l = L + 2, \dots, K, m = l + 1 \\
 & \text{or } l = K, m = 1 \\
 & \text{otherwise} \\
 0 &
 \end{cases} \quad (19),$$

1 where, as in equation (6), indices "l" and "m" range from "zero" to, and including, "K," and where

2 $s_3 = \frac{1}{2}(s_1 + s_2)$, the average of the values of the parameters s_1 and s_2 associated with the edges

3 (1,0) and (0,L+1).

4 To ensure that the directions of the tangent vectors vary continuously at and near irregular
 5 vertices, the value(s) of the sharpness parameter(s) are gradually reduced as the level of the mesh
 6 increases. In the case of smooth feature lines, the repeated update of the value of the sharpness
 7 parameter as the level of the mesh increases would provide a value that approaches zero, which, in
 8 turn, will provide that the directions of the tangent vectors vary continuously at the limit surface.
 9 In one embodiment, for a vertex v_q for which the sharpness parameters s_1 and s_2 have the same value
 10 "s" on both sides of the vertex (that is, $s_1=s_2$), the sharpness parameter update function is

$$11 \quad s(j+1) = (s(j))^2 \quad (20),$$

12 that is, $s(j+1)$, the value of the sharpness parameters s_1 and s_2 at the higher "j+1-st" subdivision level
 13 mesh, is the square of $s(j)$, the value of the sharpness parameters s_1 and s_2 at the lower ("j-the") level
 14 mesh. On the other hand, for a vertex v_q for which values of sharpness parameters s_1 and s_2 are not
 15 constant across the vertex (that is, $s_1 \neq s_2$), the sharpness parameter update functions are a function
 16 of the values of both sharpness parameters s_1 and s_2 . In one embodiment, the function is selected to
 17 be

$$18 \quad s_1(j+1) = \left(\frac{3}{4}s_1(j) + \frac{1}{4}s_2(j) \right)^2 \quad (21),$$

19 and

$$20 \quad s_2(j+1) = \left(\frac{1}{4}s_1(j) + \frac{3}{4}s_2(j) \right)^2 \quad (22),$$

21 essentially providing a linear combination corresponding to quadratic splines.

1 Since the values of the sharpness parameters s_1 and s_2 vary from level to level, the smooth
 2 curve subdivision matrix $(S_{sc,T,K,L}(s_1,s_2))_{lm}$ (equation (19)) will also not be constant from level to
 3 level. Accordingly eigen-analysis, which is conventionally used to determine positions of limit
 4 points and orientations of tangent vectors for subdivision surfaces, is not used here. As suggested
 5 above, a subdivision surface is defined by a collection limit points and normal vectors (or,
 6 equivalently, two tangent vectors for each normal vector), which, in turn, are provided by iteratively
 7 applying equation (4) from a definition level " j_D " for the mesh to the level $j=\infty$. The definition level
 8 j_D is the lowest subdivision level " j " for which the respective smooth feature line is defined in the
 9 subdivision surface. Accordingly, in order to determine the positions of the limit points, the infinite
 10 subdivision matrix

$$S_{sc,T,K,L,LP}(s_1, s_2) = \prod_{j=\infty}^{j_D} S_{sc,T,K,L}(s_1(j), s_2(j)) \quad (23),$$

11 is evaluated, where $S_{sc,T,K,L}(s_1(j), s_2(j))$ is the subdivision matrix described above in connection with
 12 equation (19) for sharpness parameters corresponding to the " j -th" level in the matrix product on the
 13 right-hand side of equation (23). For $S_{sc,T,K,L,LP}(s_1, s_2)$ on the left-hand side of equation (23),
 14 arguments s_1 and s_2 refer to the sharpness parameters at the definition level of the smooth feature
 15 line, that is, $s_1=s_1(j_D)$ and $s_2=s_2(j_D)$, and the subscript "LP" refers to "Limit Point." Since matrix
 16 multiplication is not commutative, the order of the factors in the matrix product in equation (23) is
 17 important. In equation (23), the order of the factors from left to right in the product extends from
 18 $j=\infty$ on the left to $j=j_D$ on the right, as suggested by equation (4).

19 The matrix product (equation (23)), converges to a "K+1"-by-"K+1" matrix that has "K+1"
 20 rows, with the components comprising each row being identical to the components of the other rows,
 21 in the same order. A vector of weight values l_{LP} for determining the position $\sigma(q)$ of the limit point
 22 is obtained as follows:

$$l_{LP}(s_1, s_2) = v_{LP} \cdot S_{sc,T,K,L,LP}(s_1, s_2) \quad (24),$$

1 where vector v_{LP} is a vector of weights that are used in determining the position of a limit point of
 2 a subdivision surface using the Loop subdivision methodology in the absence of a smooth feature
 3 line, that is,

4

$$v_{LP} = \left(\frac{\omega(K)}{\omega(K) + K}, \frac{1}{\omega(K) + K}, \frac{1}{\omega(K) + K}, \dots, \frac{1}{\omega(K) + K} \right) \quad (25),$$

5 where

6

$$\omega(K) = \frac{3K}{8a(K)} \quad (26),$$

7 where $a(K)$ is as defined in equation (3). The position $\sigma(q)$ of a limit point associated with a vertex
 8 $v_q(0)$ is computed from the position $c^{j_D}(q)$ of the vertex $v_q(0)$ and the positions $c^{j_D}(k)$, $k=1, \dots, K$,
 9 of the neighboring points $v_q(k)$ around the vertex $v_q(0)$, in the mesh corresponding to the definition
 10 level j_D of the smooth feature line in the subdivision surface, or to any subdivision level higher than
 11 the definition level, using the components of l_{LP} as weight values. The components of limit point
 12 weight vector $l_{LP}(s_1, s_2)$ can be satisfactorily approximated by second degree polynomials in s_1 and
 13 s_2 using six values per component and per valence.

14 If $L = \frac{K}{2}$, symmetry considerations allow the number of values per component to be

15 reduced. In that case, the components of limit point weight vector l_{LP} satisfy the following symmetry
 16 relations:

17 1. The "i-th" component of limit point weight vector l_{LP} equals the "K+2-i-th"
 18 component of l_{LP} , that is, $(l_{LP})_i = (l_{LP})_{K+2-i}$, for $i = \frac{K}{2} + 2, \dots, K$. This is a
 19 result of the fact that the subdivision matrix (equation (19)) is invariant upon

1 reflection with respect to the crease line. As a result, approximations need only be

2 generated for components $i = 0, 1, \dots, \frac{K}{2} + 1$.

3 2. Further, since the subdivision matrix (equation (19)) is also symmetric in s_1 and s_2 ,

4 $(l_{LP}(s_1, s_2))_i = (l_{LP}(s_2, s_1))_{\frac{K}{2}+2-i}$, and so approximations need only be generated

5 for $i = 0, 1, \dots, \left\lfloor \frac{K}{4} + 1 \right\rfloor$, where " $\left\lfloor \frac{x}{y} \right\rfloor$ " refers to the greatest integer in the quotient

6 " $\frac{x}{y}$;"

7 3. Further, since, if the particular values of the parameters s_1 and s_2 were interchanged,

8 the vertex v_q would behave in the same way, $(l_{LP}(s_1, s_2))_0 = (l_{LP}(s_2, s_1))_0$, and

9 so the approximation may be constructed using only symmetric, second-degree
10 polynomials in s_1 and s_2 , that is, using four coefficients; and

11 4. Further, if "K" is a multiple of "four," then, from points (1) and (2) above,

12 $(l_{LP}(s_1, s_2))_{\frac{K}{4}+1} = (l_{LP}(s_2, s_1))_{\frac{K}{4}+1}$. Accordingly, these components can be

13 approximated using symmetric second-degree polynomials using four coefficients.

1 In one embodiment, regardless of whether $L = \frac{K}{2}$, the approximating polynomials are computed

2 using a least-squares Chebyshev approximation methodology. The matrix product (equation (23))
3 is evaluated at the points of a grid in the (s_1, s_2) region made up of roots of Chebyshev polynomials

4

$$(s_1, s_2) = \left(\cos\left(\frac{\left(i + \frac{1}{2}\right)\pi}{N}\right), \cos\left(\frac{\left(j + \frac{1}{2}\right)\pi}{N}\right) \right) \quad (27),$$

5 for $i, j = 0, \dots, N-1$. In one embodiment, "N" is taken to be "eight," so that there are "sixty-four" sample
6 points in equation (27). The coefficients b_{ij} ($j = 0, \dots, 3$ in the symmetric case, or $j = 0, \dots, 5$ in the
7 asymmetric case) for the polynomial

8

$$(I_{LP})_i \approx b_{i0} + b_{i1}(s_1 + s_2) + b_{i2}(s_1^2 + s_2^2) + b_{i3}s_1s_2 \quad (28),$$

9 in the symmetric case ($i = 0$ or $\frac{K}{4} + 1$), or the polynomial

10

$$(I_{LP})_i \approx b_{i0} + b_{i1}s_1 + b_{i2}s_2 + b_{i3}s_1^2 + b_{i4}s_2^2 + b_{i5}s_1s_2 \quad (29),$$

11 in the asymmetric case ("i" otherwise), are determined by a least squares methodology. In that
12 operation, the computer graphics system 10 computes the matrix product (equation (23) evaluated
13 at points (s_1, s_2) as defined in equation (27)). Thereafter, a set of N^2 polynomials (equation (28)
14 and/or (29)) are generated using the values of s_1 and s_2 at the respective points (equation (27)) in the
15 (s_1, s_2) region, with each polynomial being equal to the respective "i-th" component of the row of the
16 matrix product. The computer graphics system 10 then determines the values of the coefficients b_{ij}

1 in the polynomials using the least squares approximation methodology. After the values of the
 2 coefficients b_{ij} have been determined, they are used to generate the values of the components $(l_{LP})_i$
 3 of limit point weight vector l_{LP} using equations (28) and (29).

4 The error in the least squares approximation has an oscillatory behavior over the region (s_1, s_2)
 5 and will be minimized if the amplitude of the oscillations is distributed evenly over the region. This
 6 can be accomplished by relaxing the constraints imposed by a weighted least squares methodology
 7 for points that are well-approximated and tightening the constraints for the points that are poorly
 8 approximated. In the weighted least squares methodology, after the values of the coefficients b_{ij}
 9 have been evaluated using the least squares methodology described above, the polynomial (equation
 10 (28) and/or (29)) is evaluated at each point in the (s_1, s_2) region, the value compared to the value
 11 generated for the respective component of the matrix product (equation (23)), and a weight value
 12 determined that reflects the difference therebetween. Thereafter, the weight values can be used in
 13 constraining the approximations of the values of the coefficients b_{ij} in an uneven manner.

14 FIG. 8 depicts a table of coefficients b_{ij} for $K=4, 6, \dots, 16$, with the indexing, in the form of
 15 the C programming language, $b\left[\frac{K}{2}-2\right][i][j]$, where "i" is the index of the component of limit

16 point weight vector l_{LP} , taking values in the range $i = 0, \dots, \left\lfloor \frac{K}{4} + 1 \right\rfloor$, and "j" is the index of the

17 coefficient b_{ij} of the approximating polynomial.

18 After the limit point weight vector l_{LP} associated with a vertex $v_q(0)$ has been generated as
 19 described above, it can be used along with the position $c^j(q)$ of the vertex $v_q(0)$ and positions $c^j(k)$
 20 of the points $v_q(k)$, $k=1, \dots, K$ neighboring the vertex $v_q(0)$ to determine the position $\sigma(q)$ of the limit
 21 point associated with the vertex $v_q(0)$ as follows

$$\sigma(q) = \sum_{i=0}^K (l_{LP}(s_1, s_2))_i c^j(i) \quad (30),$$

1

2 where, as suggested above, $i \in q \cup N(q, j)$ (reference FIG. 7), and where $(l_{LP}(s_1, s_2))$, refers to the
 3 "i-th" component of $l_{LP}(s_1, s_2)$, which, in turn, corresponds to the limit point weight vector l_{LP} for
 4 sharpness parameter values s_1 and s_2 at the definition level $j=j_D$ for the smooth feature line or at any
 5 subdivision level higher than the definition level j_D , and indices $0, \dots, K$ are as shown in FIG. 7.

6 One embodiment makes use of a similar polynomial approximation methodology to obtain
 7 good approximations for vectors l_C and l_S of weight values that are to be used in generating the
 8 tangent vectors, tangent vector weight vector l_C being used in generating the tangent vector along
 9 the smooth feature line and tangent vector weight vector l_S being used in generating the tangent
 10 vector across the smooth feature line at the limit point associated with the vertex $v_q(0)$ with which
 11 the respective tangent vector weight vectors is associated. In that methodology, a vector $l_C(J)$ is
 12 generated as follows:

$$l_C(J) = (0, 1, 0, \dots, -1, 0, \dots) \cdot \prod_{j=J}^{j_D} S_{sc, T, K, L}(s_1(j), s_2(j)) \quad (31),$$

14 where the two non-zero components of the row vector on the right hand side are a "one" at position
 15 "one" in the row vector, and a "negative one" at position $\frac{K}{2} + 1$, and " \cdot " refers to the dot product.

16 After generating vector $l_c(J)$ according to equation (31), the value is normalized and the limit taken
 17 as $J \rightarrow \infty$:

$$l_C = \lim_{j \rightarrow \infty} \frac{l_C(J)}{\|l_C(J)\|} \quad (32),$$

1 where $\|v\| = \sqrt{\sum_i v_i^2}$, that is, the Euclidean norm. The components of vector l_C (equation (32))

2 form a sequence of weight values that are used in multiplying the respective positions of the vertex
 3 $v_q(0)$ and the points $v_q(k)$, $k=1, \dots, K$, around the vertex $v_q(0)$, as shown in FIG. 7, the sum of which
 4 defines the tangent vector along the smooth feature line at the point on the subdivision surface
 5 associated with the vertex $v_q(0)$.

6 As with the components of limit point weight vector l_{LP} , the components of tangent vector
 7 weight vector l_C can be approximated by polynomials in s_1 and s_2 , in this case third degree
 8 polynomials. Accordingly, the approximations of the components of tangent vector weight vector
 9 l_C can be efficiently generated by using up to ten numbers per component and per valence. If

10 $L = \frac{K}{2}$, symmetry considerations allow the number of polynomial coefficients that are used in

11 generating components of tangent vector weight vector l_C to be reduced. In that case, the
 12 components of tangent vector weight vector l_C satisfy the following symmetry relations:

13 1. The "i-th" component and the "k+2-i-th" component of tangent vector weight vector

14 l_C are equal, that is, $(l_C)_i = (l_C)_{K+2-i}$, for $i = \frac{K}{2} + 2, \dots, K$. This is a result of the

15 invariance of the subdivision matrix (equation (19)) upon reflection with respect to
 16 the crease line. As a result, approximations need only be generated for components

17 $i = 0, 1, \dots, \frac{K}{2} + 1;$

18 2. Further, since $(l_C(s_1, s_2))_i = - (l_C(s_2, s_1))_{\frac{K}{2}+2-i}$, approximations need only be

19 generated for components $i = 0, 1, \dots, \left\lfloor \frac{K}{4} + 1 \right\rfloor$;

1 3. Further, since, if the particular values of the parameters s_1 and s_2 were interchanged,
 2 the vertex v_q would behave in the same manner except that the orientation of the
 3 tangent vector would be reversed, $(l_C(s_1, s_2))_0 = - (l_C(s_2, s_1))_0$, in which case

4 the approximation may be constructed using only anti-symmetric, third-degree
 5 polynomials in s_1 and s_2 , using four coefficients; and

6 4. Further, if K is a multiple of "four," then, from points (1) and (2) immediately above,

$$(l_C(s_1, s_2))_{\frac{K}{4}+1} = - (l_C(s_2, s_1))_{\frac{K}{4}+1}. \text{ Accordingly, these components can be}$$

7 approximated using anti-symmetric polynomials using four coefficients.

8 9. In one embodiment, the approximating polynomials are also computed using a least squares
 10 Chebyshev approximation methodology similar to that used in connection with the limit point
 11 approximation described above. The matrix product of equation (31) is evaluated at points of a grid
 12 in the (s_1, s_2) region made up of roots of Chebyshev polynomials (equation (27)). The coefficients
 13 b_{ij} ($j=0, \dots, 3$, in the anti-symmetric case, or $j=0, \dots, 9$, in the non-symmetric case) for the polynomial

$$(l_C)_i = b_{i0}(s_1 - s_2) + b_{i1}(s_1^2 - s_2^2) + b_{i2}(s_1^3 - s_2^3) + b_{i3}(s_1^2 s_2 - s_1 s_2^2) \quad (33),$$

14 15. in the anti-symmetric case ($i = 0$ or $\frac{K}{4} + 1$), or the polynomial

$$(l_C)_i = b_{i0} + b_{i1}s_1 + b_{i2}s_2 + b_{i3}s_1^2 + b_{i4}s_1s_2 + b_{i5}s_2^2 + b_{i6}s_1^3 + b_{i7}s_1^2s_2 + b_{i8}s_1s_2^2 + b_{i9}s_2^3 \quad (34),$$

16 17. in the non-symmetric case ("i" otherwise), are determined using the least squares methodology as
 18 described above.

1 In a manner similar to the generation of the tangent vector along the smooth feature line
 2 (reference equations (31) through (34) above), a vector l_s of weight values is used in generating the
 3 tangent vector across the smooth feature line. The components of tangent vector weight vector l_s
 4 comprise a sequence of weight values that are used to multiply the positions of the vertex $v_q(0)$ and
 5 points $v_q(1)$ through $v_q(K)$ around vertex $v_q(0)$ (reference FIG. 7), the sum of which comprises the
 6 tangent vector across the smooth feature line. In that methodology, a vector $l_s(J)$ is generated as
 7 follows:

$$8 \quad l_s(J) = \left(0, \sin \frac{2\pi(0)}{K}, \sin \frac{2\pi(1)}{K}, \dots, \sin \frac{2\pi(K-1)}{K} \right) \cdot \prod_{j=J}^{J_D} S_{sc,T,K,L}(s_1(j), s_2(j)) \quad (35).$$

9 Vector $l_s(J)$ is normalized and the limit taken as $J \rightarrow \infty$, in the same manner as described above in
 10 connection with equation (32), to provide tangent vector weight vector l_s .

11 When $L = \frac{K}{2}$, tangent vector weight vector l_s is

$$12 \quad l_s = \left(0, \sin \frac{2\pi(0)}{K}, \sin \frac{2\pi(1)}{K}, \dots, \sin \frac{2\pi(K-1)}{K} \right) \quad (36),$$

13 which is the same as in the case of a smooth feature line associated with a sharpness parameter of
 14 zero.

15 The components of tangent vector weight vector l_s can be approximated as $l_s(s_1, s_2)$ using
 16 third degree polynomials in sharpness parameters s_1 and s_2 . The operations performed in generating
 17 the coefficients for the polynomials correspond to the operations performed in generating the
 18 coefficients for the polynomials described above for the tangent vector weight vector l_c used in
 19 generating the tangent vector along the smooth feature line (equations (31) through (34)).

1 FIG. 9 depicts a table of coefficients b_{ij} for $K=4, 6, \dots, 12$, with the indexing, in the form of

2 the C programming language, $b\left[\frac{K}{2}-2\right][i][j]$, where "i" is the index of the component of l_C ,

3 taking values in the range $i = 0, \dots, \left\lfloor \frac{K}{4} + 1 \right\rfloor$, and "j" is the index of the coefficient b_{ij} of the

4 approximating polynomial.

5 As noted above, the tangent vector weight vectors $l_C (= l_C(s_1, s_2))$ and $l_S (= l_S(s_1, s_2))$ (where s_1 and
 6 s_2 are the sharpness parameter values at the definition level $j=j_D$ for the smooth feature line) are used,
 7 along with the position $c^j(q)$ of the vertex $v_q(0)$ and the positions $c^j(k)$ of the neighboring points
 8 $v_q(k)$, $k=1, \dots, K$. In particular, the components of the respective tangent vector weight vectors are
 9 used to weight the positions of the vertex and neighboring points, with the sum comprising the
 10 tangent vector. That is, the tangent vector $e_C(q)$ along the smooth feature line and the tangent vector
 11 $e_S(q)$ across the smooth feature line are generated as

$$12 e_C(q) = \sum_{i=0}^K \left(l_C(s_1, s_2) \right)_i c^j(i) \quad (37),$$

$$e_S(q) = \sum_{i=0}^K \left(l_S(s_1, s_2) \right)_i c^j(i)$$

13 where $l_C(s_1, s_2) = l_C$ and $l_S(s_1, s_2) = l_S$ (where s_1 and s_2 are the sharpness parameter values at the definition
 14 level $j=j_D$ for the smooth feature line), and indices $0, \dots, K$ are as shown in FIG. 7.

15 In some circumstances, it is impractical to use polynomials to generate approximations for
 16 limit point weight vector l_{LP} and tangent vector weight vectors l_C and l_S . For example, if the number
 17 of configurations of smooth feature lines, each requiring a different set of polynomials, exceeds
 18 some number, the number of polynomials would increase to an extent that it would be impractical
 19 to use that methodology. An alternative methodology makes use of extrapolation to generate
 20 approximations for limit point weight vector l_{LP} and tangent vector weight vectors l_C and l_S .

1 The extrapolation methodology for generating an approximation of limit point weight vector
 2 $l_{LP}(s_1, s_2)$ starts with the first few factors of equation (23) for finite values of index "j." In the
 3 following, it will be assumed that the definition level j_D for the smooth feature line is level "zero,"
 4 although it will be appreciated that this will not restrict the generality of the description. The matrix
 5 product generated by taking factors $j=J, J-1, \dots, 0$ is given by

6

$$S_{sc,T,K,L,Lp}(J)(s_1, s_2) := \prod_{j=J}^0 S_{sc,T,K,L}(s_1(j), s_2(j)) \quad (38),$$

7 with

8

$$S_{sc,T,K,L,Lp}(0)(s_1, s_2) := I_{K+1} \quad (39),$$

9 where, for the limit case $J=0$ (in which case there would be no factors in generating the product
 10 matrix), the product matrix $S_{sc,T,K,L,Lp}(0)(s_1, s_2)$ is the identity matrix of size "K+1" by "K+1." A
 11 vector of weight values l_{LP} for determining the position $\sigma(q)$ of the limit point is obtained as follows:

12

$$l_{LP}(J)(s_1, s_2) = v_{LP} \cdot S_{sc,T,K,L,Lp}(J)(s_1, s_2) \quad (40),$$

13 where v_{LP} is as defined above in connection with equations (28) and (29).

14 In one embodiment, a reasonably accurate approximation of limit point weight vector l_{LP} can
 15 be obtained using just the vectors corresponding to $J=0, 1, 2, 3$ by using polynomial extrapolation.
 16 The convergence of $l_{LP}(J)(s_1, s_2)$ to $l_{LP}(s_1, s_2)$ is very rapid, with the error $\epsilon(J) = |l_{LP}(J) - l_{LP}|$
 17 approaching zero as 2^{-2^J} if the update rule described above in connection with equations (20)
 18 through (22) are used for the sharpness parameters. The approximation to l_{LP} is constructed by
 19 generating the third degree polynomial that interpolates the points $\{x = 2^{-2^J}, y = l_{LP}(J)\}, J=0,$
 20 1, 2, 3, and then evaluating this polynomial by extrapolation at the point $x=0$, that is

1

$$l_{LP} \approx \sum_{J=0}^3 b_J l_{LP}(J) \quad (41),$$

2 where $l_{LP}(J)$ is as defined in equation (40) and where the coefficients b_j are given by

3

$$b_0 = \frac{-135}{120015}$$

$$b_1 = \frac{1270}{120015} \quad (42).$$

$$b_2 = \frac{-12192}{120015}$$

$$b_3 = \frac{131072}{120015}$$

4 This extrapolation methodology can be used for any vertex, whether the vertex is regular or irregular.
 5 In addition, the methodology can be used for any configuration of creases through the respective
 6 vertex, and any value of valence "K" for the respective vertex.

7 An extrapolation methodology can also be used to generate the tangent vector weight vectors
 8 l_C and l_S . Generally, in that methodology the matrix products $S_{sc,T,K,L,LP}(J)(s_1, s_2)$ described above
 9 (equations (38) and (39)), which are then

10 (1) multiplied by a vector v_C , if tangent vector weight vector l_C is being generated, or a
 11 vector v_S , if tangent vector weight vector l_S is being generated, and

12 (2) the result from (1) is thereafter multiplied by a dilation factor that maintains the
 13 respective tangent vector weight vector in normalized form.

14 The approximations of the tangent vector weight vectors l_C and l_S are generated from

$$l_C(J)(s_1, s_2) = d(K)^J v_C \cdot S_{sc,T,K,L,LP}(J)(s_1, s_2) \quad (43),$$

$$l_S(J)(s_1, s_2) = d(K)^J v_S \cdot S_{sc,T,K,L,LP}(J)(s_1, s_2)$$

where $d(K)$ represents the dilation factor. The vectors v_C and v_S are given by

$$v_C = \left(0, \cos \frac{2\pi(0)}{K}, \cos \frac{2\pi(1)}{K}, \dots, \cos \frac{2\pi(K-1)}{K} \right) \quad (44),$$

$$v_S = \left(0, \sin \frac{2\pi(0)}{K}, \sin \frac{2\pi(1)}{K}, \dots, \sin \frac{2\pi(K-1)}{K} \right)$$

and the dilation factor $d(K)$ is the reciprocal of one of the eigenvalues of the subdivision matrix for the smooth case, namely,

$$d(K) = \frac{1}{\frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{K}} \quad (45).$$

Approximations to the tangent vector weight vectors l_C and l_S are given by

$$l_C = \sum_{J=0}^3 b_J l_C(J) \quad (46),$$

$$l_S = \sum_{J=0}^3 b_J l_S(J)$$

where the coefficients b_J have the form as given in equation (42). This extrapolation methodology can be used in connection with all configurations. The tangent vector generated using the weight vector l_C will be oriented approximately in the direction of the edge $[1,0]$ between vertex $v_q(0)$ and point $v_q(1)$, and the tangent vector generated using the weight vector l_S will be oriented

1 approximately perpendicular to the edge [1,0]. Although the angle between edges [1,0] and [0,L+1]
 2 for a given level "j" in a subdivision surface is typically not one hundred and eighty degrees, as the
 3 level "j" approaches infinity, the angle will approach one hundred and eighty degrees. Accordingly,
 4 in the limit, that is, as "j" approaches infinity, the tangent vector along the edge on one side of the
 5 limit point associated with a vertex $v_q(0)$ will approach the opposite direction as the tangent vector
 6 along the edge on the other side of the limit point associated with the same vertex. Similarly, the
 7 tangent vector across the edge on one side of the limit point associated with the vertex $v_q(0)$ will
 8 approach the opposite direction as the tangent vector across the edge on the other side of the limit
 9 point associated with the same vertex.

10 With this background, operations performed by the computer graphics system 10 in
 11 generating the weight vectors l_{LP} , l_C and l_S useful in generating the limit point and tangent vectors
 12 for a subdivision surface for a triangular mesh will be described in connection with the flowchart
 13 depicted in FIG. 10. With reference to that FIG., the computer graphics system 10 first initializes
 14 the subdivision matrices $S_{sc,T,K,L}(s_1(j),s_2(j))$, $j=0,1,2$ (step 100) as described above in connection with
 15 equation (19). Thereafter, the computer graphics system 10 generates the matrix products
 16 $S_{sc,T,K,L,LP}(J)(s_1,s_2)$ for $J=2,3$ (step 101). For $J=1$, $S_{sc,T,K,L,LP}(1)(s_1,s_2)=S_{sc,T,K,L}(s_1,s_2)$, and, as indicated
 17 above, for $J=0$, $S_{sc,T,K,L,LP}(0)(s_1,s_2)$ is the "K+1"-by-"K+1" identity matrix.

18 After the computer graphics system 10 has generated the matrix products $S_{sc,T,K,L,LP}$, it takes
 19 the first row of each matrix product $S_{sc,T,K,L,LP}(J)(s_1,s_2)$, $J=0, 1, 2, 3$, and generates component-wise
 20 an approximation to limit point weight vector l_{LP} in accordance with the extrapolation formula in
 21 equation (41) (step 102). In addition, the computer graphics system 10 uses the matrix products
 22 $S_{sc,T,K,L,LP}(J)(s_1,s_2)$, dilation factor $d(K)$ and vectors v_C and v_S to generate the vectors $l_C(J)$ and $l_S(J)$,
 23 $J=1, 2$, and 3 as described above in connection with equation (43) (step 103). For $J=0$, the respective
 24 weight vectors are $l_C(0)=v_C$ and $l_S(0)=v_S$. Following step 103, the computer graphics system 10 can
 25 use $l_C(J)$ and $l_S(J)$ to generate the approximations to the tangent vector weight vectors l_C and l_S in
 26 accordance with equation (46) (step 104). After the limit point weight vector l_{LP} and tangent vector
 27 weight vectors l_C and l_S have been generated, the limit point and tangent vectors can be generated
 28 as described above (reference equations (30) and (37), respectively), and the normal vector can also
 29 be generated as the cross product between the tangent vectors (step 105).

Operations performed by the computer graphics system 10 in connection with generating a smooth feature line in the case of a subdivision surface defined by a quadrilateral mesh will be described in connection with FIGS. 11 through 13 and the flow chart in FIG. 10. With reference to FIG. 11, that FIG. depicts a portion of a subdivision surface defined by a triangular mesh with vertex v_q (vertex $v_q=v_q(0)$) and points $v_q(1)$ through $v_q(2K)$ with indices numbered as depicted in the FIG. It will be appreciated that points $v_q(1)$ through $v_q(K)$ are first-order neighbors of vertex v_q , and points $v_q(K+1)$ through $v_q(2K)$ are second-order neighbors of vertex v_q . The vertices are labeled such that a crease passes through point $v_q(1)$, vertex $v_q(0)$ and point $v_q(L+1)$ along edges $v_q(1),v_q$ (which will be referred to as edge (1,0)) and $v_q,v_q(L+1)$ (which will be referred to as edge (0,L+1)). The value of the sharpness parameter associated with the edge (1,0) will be referred to as s_1 and the value of the sharpness parameter associated with the edge (0,L+1) will be referred to as s_2 . Generally the subdivision rule described in equations (13) and (14) can be used to determine the positions of respective vertices, using, instead of the subdivision matrix described above in connection with equation (15), the subdivision matrix

$$\left(S_{sc,Q,K,L}(s_1, s_2) \right)_{l,m} = \begin{cases}
\left(1 - s_3\right) \left(1 - \frac{7}{4K}\right) + \frac{3}{4} s_3 & \text{if } l = 0, m = 0 \\
\left(1 - s_3\right) \left(\frac{3}{2K^2}\right) + \frac{1}{8} s_3 & \text{if } l = 0, m = 1 \text{ or } L + 1 \\
\left(1 - s_3\right) \left(\frac{3}{2K^2}\right) & \text{if } l = 0, m = 2, \dots, L \\
& \text{or } l = 0, m = L + 2, \dots, K \\
\left(1 - s_3\right) \left(\frac{1}{4K^2}\right) & \text{if } l = 0, m = K + 1, \dots, 2K \\
\frac{3}{8} \left(1 - s_2\right) + \frac{1}{2} s_2 & \text{if } l = 1, m = 0 \text{ or } 1 \\
\frac{1}{16} \left(1 - s_2\right) & \text{if } l = L + 1, m = 2, K, \\
& \quad K + 1 \text{ or } 2K \\
\frac{3}{8} \left(1 - s_1\right) + \frac{1}{2} s_1 & \text{if } l = L + 1, m = 0 \text{ or } L + 1 \\
\frac{1}{16} \left(1 - s_1\right) & \text{if } l = L + 1, m = L, L + 2, \\
& \quad K + 1 \text{ or } K + L + 1 \\
\frac{3}{8} & \text{if } l = 2, \dots, L, m = 0 \\
& \text{or } l = L + 2, \dots, K, m = 0 \\
& \text{or } l = m = 2, \dots, L \\
& \text{or } l = m = L + 2, \dots, K \\
\frac{1}{16} & \text{if } l = 2, \dots, L, m = l - 1 \\
& \text{or } l = 2, \dots, L, m = l + 1 \\
& \text{or } l = 2, \dots, L, m = K + l - 1 \\
& \text{or } l = 2, \dots, L, m = K + l \\
& \text{or } l = L + 2, \dots, K, m = l - 1 \\
& \text{or } l = L + 2, \dots, K - 1, m = l + 1 \\
& \text{or } l = K, m = 1 \\
& \text{or } l = L + 2, \dots, K, m = K + l - 1 \\
& \text{or } l = L + 2, \dots, K, m = K + l \\
& \text{if } l = K + 1, \dots, 2K - 1, m = 0, \\
& \quad l - K, l - K + 1 \text{ or } l \\
& \text{or } l = 2K, m = 0, K, 1 \text{ or } 2K \\
0 & \text{otherwise}
\end{cases} \quad (47),$$

1 is used, where, as with equation (19), $s_3 = \frac{1}{2}(s_1 + s_2)$, the average of the values of the parameters
 2 s_1 and s_2 associated with the edges (1,0) and (0,L+1).

3 As with the case of a subdivision surface defined by a triangular mesh, to ensure that the
 4 direction of the respective tangent vectors vary continuously at and near irregular vertices at the
 5 various subdivision levels, the value(s) of the sharpness parameter(s) are gradually reduced as the
 6 subdivision level of the mesh increases. In one embodiment, the computer graphics system varies
 7 the value(s) of the sharpness parameter(s) in the case of a subdivision surface defined by a
 8 quadrilateral mesh in the same manner as in the case of a subdivision surface defined by a triangular
 9 mesh (reference equations (20) through (22) above).

10 As with the case of a subdivision surface defined by a triangular mesh, to determine the
 11 positions of the limit point associated with a vertex $v_q(0)$ in the case of a subdivision surface defined
 12 by a quadrilateral mesh, the infinite subdivision matrix product

$$3 S_{sc,Q,K,L,Lp}(s_1, s_2) = \prod_{j=\infty}^{j_D} S_{sc,Q,K,L}(s_1(j), s_2(j)) \quad (48)$$

14 is evaluated, where $S_{sc,Q,K,L}(s_1(j), s_2(j))$ is the subdivision matrix from equation (47) for sharpness
 15 parameter values s_1 and s_2 associated with the j -th level and index j_D refers to the definition level of
 16 the smooth crease. In the left-hand side of equation (48), for the arguments s_1 and s_2 , $s_1=s_1(j_D)$ and
 17 $s_2=s_2(j_D)$. As in the case of a subdivision surface defined by a triangular mesh, the matrix product
 18 in equation (48) converges to a matrix with identical rows, in this case each row being of size $2K+1$.
 19 A vector of weight values l_{lp} for determining the position $\sigma(q)$ of a limit point associated with vertex
 20 $v_q(0)$ is obtained as follows:

$$21 l_{lp}(s_1, s_2) = v_{LP} \cdot S_{sc,Q,K,L,Lp}(s_1, s_2) \quad (49),$$

22 where vector v_{LP} is a vector of weights that are used in determining the position of a limit point of
 23 a subdivision surface defined by a quadrilateral mesh in the absence of a smooth feature line, that
 24 is,

$$25 v_{LP} = \frac{1}{K(K+5)}(K^2, 4, \dots, 4, 1, \dots, 1) \quad (50),$$

1 where, in equation (50), there are "K" components having the value "four" and "K" components with
 2 value "one."

3 The components $l_{LP}(s_1, s_2)_i$, $i=0, \dots, 2K$, of the row vector $l_{LP}(s_1, s_2)$, and the positions $c^j(0)$ of
 4 the vertex $v_q(0)$ and $c^j(i)$ ($i=1, \dots, 2K$) of the neighboring points $v_q(i)$ are used to determine the position
 5 $\sigma(q)$ of a limit point associated with the vertex $v_q(0)$ as follows

$$6 \quad \sigma(q) = \sum_{i=0}^{2K} \left(l_{LP}(s_1, s_2) \right)_i c^j(i) \quad (51),$$

7 where, as suggested above, $i \in q \cup N_e(q, j) \cup N_f(q, j)$ (reference FIG. 11).

8 As with the case of a subdivision surface defined by a triangular mesh, the components of
 9 the limit point weight vector $l_{LP}(s_1, s_2)$ can be approximated using polynomial expansions. The
 10 components of limit point weight vector $l_{LP}(s_1, s_2)$ can be satisfactorily approximated by second
 11 degree polynomials in s_1 and s_2 using six values per component and per valence. If the value of "K"
 12 is even, and if $L = \frac{K}{2}$, symmetry considerations allow the number of values per component to be

13 reduced. In that case, the components of limit point weight vector l_{LP} satisfy the following symmetry
 14 relations:

15 1. the "i-th" component of limit point weight vector $l_{LP}(s_1, s_2)$ equals the "K+2-i-th"
 16 component of limit point weight vector $l_{LP}(s_1, s_2)$, that is,

$$17 \quad \left(l_{LP}(s_1, s_2) \right)_i = \left(l_{LP}(s_1, s_2) \right)_{K+2-i}, \text{ for } i = 2, \dots, \frac{K}{2}, \text{ and the "i-th" component of}$$

18 limit point weight vector $l_{LP}(s_1, s_2)$ equals the "3K+1-i-th" component of limit point
 19 weight vector $l_{LP}(s_1, s_2)$, that is, $\left(l_{LP}(s_1, s_2) \right)_i = \left(l_{LP}(s_1, s_2) \right)_{3K+1-i}$, for

$$20 \quad i = K+1, \dots, K+\frac{K}{2};$$

1 2. $\left(l_{LP}(s_1, s_2)\right)_i = \left(l_{LP}(s_1, s_2)\right)_{\frac{K}{2}+2-i}, \quad \text{f o r } \quad i = 1, \dots, \frac{K}{2} + 1, \quad \text{a n d}$

2 $\left(l_{LP}(s_1, s_2)\right)_i = \left(l_{LP}(s_1, s_2)\right)_{2K+\frac{K}{2}+1-i}, \text{ for } i = K+1, \dots, K+\frac{K}{2}+1;$

3 3. $\left(l_{LP}(s_1, s_2)\right)_0 = \left(l_{LP}(s_2, s_1)\right)_0; \text{ and}$

4 4. $\left(l_{LP}(s_1, s_2)\right)_{\frac{K}{4}+1} = \left(l_{LP}(s_2, s_1)\right)_{\frac{K}{4}+1}, \text{ if "K" is a multiple of "four," and}$

5 $\left(l_{LP}(s_1, s_2)\right)_{K+\left\lfloor \frac{K}{4} \right\rfloor+1} = \left(l_{LP}(s_2, s_1)\right)_{K+\left\lfloor \frac{K}{4} \right\rfloor+1}, \text{ if "K" is not a multiple of "four."}$

6 Given the symmetry relations described immediately above, it is only necessary to generate

7 approximations for limit point weight vector l_{LP} for $i = 0, \dots, \left\lfloor \frac{K}{4} \right\rfloor + 1$ and

8 $i = K+1, \dots, K+\left\lfloor \frac{K}{4} \right\rfloor + 1$, and, in addition, symmetric polynomials can be used for $i=0$ for any

9 even value of "K," as well as for $i = \frac{K}{4} + 1$ if "K" is a multiple of "four," or $i = K+\left\lfloor \frac{K}{4} \right\rfloor + 1$ if

10 "K" is even but not a multiple of "four."

11 As in the case of a subdivision surface defined by a triangular mesh, in one embodiment in the
12 case of a subdivision surface defined by a quadrilateral mesh, regardless of whether $L = \frac{K}{2}$, the

13 approximating polynomials are computed using a least-squares Chebyshev approximation
14 methodology (reference equation (27) above). In this case, the coefficients b_{ij} ($j=0, \dots, 3$ in the
15 symmetric case, or $j=0, \dots, 5$ in the asymmetric case) for the polynomial

$$\left(I_{LP} \right)_i \approx b_{i0} + b_{i1} (s_1 + s_2) + b_{i2} (s_1^2 + s_2^2) + b_{i3} s_1 s_2 \quad (52),$$

in the symmetric case, or the polynomial

$$\left(l_{LP}\right)_i \approx b_{i0} + b_{i1}s_1 + b_{i2}s_2 + b_{i3}s_1^2 + b_{i4}s_1s_2 + b_{i5}s_2^2 \quad (53),$$

in the asymmetric case, are determined by a least squares methodology. FIG. 12 depicts a table of coefficients b_{ij} for $K=4$, $L=2$, for values of "i" $i=0; 1; 2; 5$. After the coefficients b_{ij} have been generated (equations (52) and (53)), they are used to generate the limit point weight vector l_{LP} in the same manner as in the case of a subdivision surface defined by a triangular mesh, which can be used to generate the position of the limit point $\sigma(q)$ as described above in connection with equation (51).

The tangent vectors in the case of a subdivision surface defined by a quadrilateral mesh are generated in a manner similar to that used to generate tangent vectors in the case of a subdivision surface defined by a triangular mesh. Generally, in that methodology the matrix products $S_{sc,Q,K,L,P}(J)(s_1, s_2)$ as described above (equation (48)) are generated, which are then

- (1) multiplied by a vector v_C , if tangent vector weight vector l_C is being generated, or a vector v_S , if tangent vector weight vector l_S is being generated, and
- (2) the result from (1) is thereafter multiplied by a dilation factor that maintains the respective tangent vector in normalized form,

as in the case of the tangent vector weight vectors in the case of a triangular mesh. Initially, the vectors $l_C(J)(s_1, s_2)$ and $l_S(J)(s_1, s_2)$ are generated as

$$l_C(s_1, s_2) = d(K)^J v_C \cdot S_{sc, Q, K, L, LP}(J)(s_1, s_2) \quad (54).$$

$$l_S(s_1, s_2) = d(K)^J v_s \cdot S_{sc, O, K, L, LP}(J)(s_1, s_2)$$

In the case of a subdivision surface defined by a quadrilateral mesh, the respective vectors v_C and v_S are given by

$$(v_C)_i = \begin{cases} 0 & \text{if } i = 0 \\ A_K \cos \frac{2\pi(i-1)}{K} & \text{if } i = 1, \dots, K \\ \cos \frac{2\pi(i-K-1)}{K} + \cos \frac{2\pi(i-K)}{K} & \text{if } i = K+1, \dots, 2K \end{cases} \quad (55),$$

$$(v_S)_i = \begin{cases} 0 & \text{if } i = 0 \\ A_K \sin \frac{2\pi(i-1)}{K} & \text{if } i = 1, \dots, K \\ \sin \frac{2\pi(i-K-1)}{K} + \sin \frac{2\pi(i-K)}{K} & \text{if } i = K+1, \dots, 2K \end{cases}$$

and the dilation factor is given as the reciprocal of the subdominant eigenvalue of the subdivision matrix (equation 47) for a smooth interior vertex:

$$d(K) = \frac{1}{\frac{A_K}{16} + \frac{1}{4}} \quad (56),$$

where A_K is defined by

$$A_K = 1 + \cos\left(\frac{2\pi}{K}\right) + \cos\left(\frac{\pi}{K}\right) \sqrt{2\left(9 + \cos\frac{2\pi}{K}\right)} \quad (57).$$

The tangent vector weight vectors l_C and l_S are used to generate the tangent vectors as follows:

$$e_C(q) = \sum_{i=0}^{2K} \left(l_C(s_1, s_2) \right)_i c^J(i) \quad (58),$$

$$e_S(q) = \sum_{i=0}^{2K} \left(l_S(s_1, s_2) \right)_i c^J(i)$$

1 where the indexing is as shown in FIG. 11.

2
3 As in the case of a subdivision surface defined by a triangular mesh, in the case of a
4 subdivision surface defined by a quadrilateral mesh the tangent vector $e_C(q)$ generated for a vertex
5 $v_q(0)$ generated using the tangent vector weight vector l_C will be oriented approximately along the
6 smooth feature line through the limit point associated with the vertex $v_q(0)$, and the tangent vector
7 $e_S(q)$ that is generated using the tangent vector weight vector l_S will be oriented approximately
8 perpendicular to the smooth feature line through the limit point associated with the vertex $v_q(0)$.

9
10 Also, as with the case of a subdivision surface defined by a triangular mesh, the tangent
11 vector weight vectors in the case of a subdivision surface defined by a quadrilateral mesh can be
12 approximated using polynomial approximations. The components of tangent vector weight vectors
13 $l_C(s_1, s_2)$ and $l_S(s_1, s_2)$ can be satisfactorily approximated by third degree polynomials in s_1 and s_2
14 using, in general, ten values per component and per valence. If the value of "K" is even, and
if $L = \frac{K}{2}$, symmetry considerations allow the number of values per component to be reduced. In

15 that case, the components of tangent vector weight vector l_C satisfy the following symmetry
16 relations:

17 1. the "i-th" component of tangent vector weight vector $l_C(s_1, s_2)$ equals the "K+2-i-th"
18 component of tangent vector weight vector $l_C(s_1, s_2)$, that

19 is, $\left(l_C(s_1, s_2) \right)_i = \left(l_C(s_1, s_2) \right)_{K+2-i}$, for $i = 2, \dots, \frac{K}{2}$, and the "i-th" component

20 of tangent vector weight vector $l_C(s_1, s_2)$ equals the "3K+1-i-th" component of tangent

1 vector weight vector $l_C(s_1, s_2)$, that is, $(l_C(s_1, s_2))_i = (l_C(s_1, s_2))_{3K+1-i}$, for

2 $i = K+1, \dots, K + \frac{K}{2};$

3 2. $(l_C(s_1, s_2))_i = - (l_C(s_1, s_2))_{\frac{K}{2}+2-i}, \quad \text{for } i = 1, \dots, \frac{K}{2}+1, \quad \text{and}$

4 $(l_C(s_1, s_2))_i = - (l_C(s_1, s_2))_{2K+\frac{K}{2}+1-i}, \text{ for } i = K+1, \dots, K + \frac{K}{2}+1;$

5 3. $(l_C(s_1, s_2))_0 = - (l_C(s_2, s_1))_0; \text{ and}$

6 4. $(l_C(s_1, s_2))_{\frac{K}{4}+1} = - (l_C(s_2, s_1))_{\frac{K}{4}+1}, \text{ if "K" is a multiple of "four," and}$

7 $(l_C(s_1, s_2))_{K+\left\lfloor \frac{K}{4} \right\rfloor+1} = - (l_C(s_2, s_1))_{K+\left\lfloor \frac{K}{4} \right\rfloor+1}, \text{ if "K" is not a multiple of "four."}$

8 Given the symmetry relations described immediately above, it is only necessary to generate

9 approximations for $i = 0, \dots, \left\lfloor \frac{K}{4} \right\rfloor + 1$ and $i = K+1, \dots, K + \left\lfloor \frac{K}{4} \right\rfloor + 1$, and, in addition, anti-

10 symmetric polynomials can be used for $i = 0$ for any even value of "K," as well as for $i = \frac{K}{4} + 1$

11 if "K" is a multiple of "four," or $i = K + \left\lfloor \frac{K}{4} \right\rfloor + 1$ if "K" is even but not a multiple of "four."

12 As in the case of a subdivision surface defined by a triangular mesh, in one embodiment, the
13 approximating polynomials are also computed using a least squares Chebyshev approximation
14 methodology similar to that used in connection with the limit point approximation described above.

1 The matrix product of equation (48) is evaluated at points of a grid in the (s_1, s_2) region made up of
 2 roots of Chebyshev polynomials (equation (27)). In this case, the coefficients b_{ij} ($j=0, \dots, 3$, in the
 3 anti-symmetric case, or $j=0, \dots, 9$, in the non-symmetric case) for the polynomial

4
$$(l_C)_i \approx b_{i0}(s_1 - s_2) + b_{i1}(s_1^2 - s_2^2) + b_{i2}(s_1^3 - s_2^3) + b_{i3}(s_1^2 s_2 - s_1 s_2^2) \quad (59),$$

5 in the anti-symmetric case, or the polynomial

6
$$(l_C)_i \approx b_{i0} + b_{i1}s_1 + b_{i2}s_2 + b_{i3}s_1^2 + b_{i4}s_1s_2 + \\ b_{i5}s_2^2 + b_{i6}s_1^3 + b_{i7}s_1^2s_2 + b_{i8}s_1s_2^2 + b_{i9}s_2^3 \quad (60),$$

7 in the non-symmetric case, are determined using the least squares methodology. Operations
 8 performed in connection with the least squares methodology correspond to those described above
 9 in the case of a subdivision surface defined by a triangular mesh. The components of tangent vector
 10 weight vector l_s can be approximated as $l_s(s_1, s_2)$ using third degree polynomials in sharpness
 11 parameters s_1 and s_2 . The operations performed in generating the coefficients for the polynomials
 12 correspond to the operations performed in generating the coefficients for the polynomials described
 13 above for the tangent vector weight vector l_C used in generating the tangent vector along the smooth
 14 feature line. FIG. 13 depicts a table of coefficients b_{ij} for $K=4$, $L=2$, corresponding to $i=0; 1; 2; 5$.

15 After the tangent vector weight vectors l_C and l_s have been generated, they are used to
 16 generate the tangent vectors e_C and e_s as described above in connection with equation (58).

17 As in the case of a subdivision surface defined by a triangular mesh, in some circumstances,
 18 it is impractical to use polynomials to generate approximations for limit point weight vector l_{LP} and
 19 tangent vector weight vectors l_C and l_s . An alternative methodology makes use of polynomial
 20 extrapolation to generate approximations for limit point weight vector l_{LP} and tangent vector weight
 21 vectors l_C and l_s . This extrapolation methodology is similar to the methodology described above in
 22 connection with equations 38-46.

23 Operations performed by the computer graphics system 10 in connection with generating the
 24 limit point weight vector l_{LP} and tangent vector weight vectors l_C and l_s for a subdivision surface
 25 defined by a quadrilateral mesh, using the polynomial extrapolation methodology, are similar to
 26 those described above in connection with FIG. 10 for a subdivision surface defined by a triangular
 27 mesh, and reference should be made to FIG. 10 for that description.

1 The arrangement provides a number of advantages. In particular, the invention provides an
2 arrangement for generating a subdivision surface from a representation defined by a control mesh
3 at a selected level, by applying the same parameterized subdivision rule at each level, with the
4 value(s) of the parameter(s) varying at at least some of the levels. The invention specifically
5 provides for the generation of a smooth feature line in a subdivision surface using the same
6 parameterized subdivision rule at each level, with the value(s) of the parameter(s) defining the
7 smooth feature line varying at at least some of the levels. The resulting subdivision surface is of
8 relatively high quality and can be efficiently evaluated.

9 It will be appreciated that numerous changes and modifications may be made to the
10 arrangement as described herein. For example, it will be appreciated that, although the arrangement
11 has been described in connection with meshes of triangular and quadrangular faces, it will be
12 appreciated that the arrangement may find utility in connection with meshes having faces of different
13 polygonal structures.

14 Furthermore, although specific subdivision rules (equations (19) and (46)) have been
15 described, it will be appreciated that other subdivision rules may find utility.

16 In addition, although a methodology has been described for updating the sharpness
17 parameters s_1 and s_2 , and although various methodologies have been described for approximating
18 weight vectors l_{LP} , l_C and l_S , it will be appreciated that other parameter update and approximation
19 methodologies may find utility.

20 Furthermore, in connection with the extrapolation methodologies described herein, it will be
21 appreciated that the number of coefficients b_j , and the upper limit ("three") on the summations in
22 equations (41) and (46) may differ if a polynomial other than of order "three" is used in the
23 extrapolation.

24 It will be appreciated that a system in accordance with the invention can be constructed in
25 whole or in part from special purpose hardware or a general purpose computer system, or any
26 combination thereof, any portion of which may be controlled by a suitable program. Any program
27 may in whole or in part comprise part of or be stored on the system in a conventional manner, or it
28 may in whole or in part be provided into the system over a network or other mechanism for
29 transferring information in a conventional manner. In addition, it will be appreciated that the system
30 may be operated and/or otherwise controlled by means of information provided by an operator using

1 operator input elements (not shown) which may be connected directly to the system or which may
2 transfer the information to the system over a network or other mechanism for transferring
3 information in a conventional manner.

4 The foregoing description has been limited to a specific embodiment of this invention. It will
5 be apparent, however, that various variations and modifications may be made to the invention, with
6 the attainment of some or all of the advantages of the invention. It is the object of the appended
7 claims to cover these and such other variations and modifications as come within the true spirit and
8 scope of the invention.

9 What is claimed as new and desired to be secured by Letters Patent of the United States is: